Communication Matrix

From a Communication Network

Rows -> directions data can be transmitted
Columns -> sources from which data can be received

Which computer can send data directly to the fewest points? **D**

Which computer can receive data from the most points? **A, D**
$X^2$ represents the number of routes one computer can send data to a second computer in the network through exactly one data point.

Calculate $X^2$.

\[
\begin{array}{cccc}
  & A & B & C & D \\
A & 1 & 1 & 1 & 1 \\
B & 0 & 2 & 1 & 1 \\
C & 1 & 1 & 1 & 0 \\
D & 1 & 0 & 1 & 0 \\
\end{array}
\]

The "2" in the second row and column indicates that computer B can send data to itself using one relay point in two different routes.

If Matrix $X$ represents direct communication, and matrix $X^2$ represents communication using 2 steps, what will matrix $X^3$ represent?

If I want to determine the number of routes a computer can send information to another in no more than 2 steps, $X + X^2$.

Calculate $X + X^2$
In how many ways can $A$ send a message to $B$ using no more than one relay point (two steps)?

Dominance Relations

In a tennis tournament, Adam beats Bill and Chad, but loses to Derek; Bill beats Chad and Derek; and Chad beats only Derek.

\[ M = \begin{bmatrix}
A & B & C & D \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}\]

Who has the best record?

Who has the worst?

Since Adam and Bill have both won 2 and lost 1, they are tied at this stage, and to determine a tournament winner, second-stage dominance must be determined by calculating $M^2$.

Second stage dominance means that, for example, Regis beats Mullen and Mullen beats Valor, so Regis would have second stage dominance over Mullen.

Calculate $M^2$. 

\[ M^2 = \begin{bmatrix}
A & B & C & D \\
0 & 0 & 1 & 2 \\
1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0
\end{bmatrix}\]
Notice that Adam has a second stage dominance over Chad, and a pair of second stage dominances over Derek. So, even though Adam lost to Derek, he beat two people that beat Derek.

To determine the tournament winner, calculate $M + M^2$.

\[
M + M^2 = \begin{bmatrix}
0 & 1 & 2 & 2 \\
1 & 0 & 1 & 2 \\
1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 \\
\end{bmatrix}
\]

Try this:

A communication network for 4 ships is shown.

Write the communication matrix.

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
\end{bmatrix}
\]
Which ship can send messages to the greatest number of other ships?

\[ C \]

Write the matrix that represents the number of routes messages can be sent from one ship to another using exactly one relay.

\[ X^2 = A \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \]

How many ways can a message be sent from Ship C to Ship B using no more than one relay?

\[ X + X^2 = A \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 3 & 2 & 2 \\ 1 & 2 & 2 & 2 \end{bmatrix} \]

In a tennis tournament, Jane beats Cindy, but loses to Yoko and Anna. Cindy beats Yoko and Anna. Yoko beats Jane and Anna. Anna beats only Jane.

Write the dominance matrix and the matrix showing second stage dominances.

\[ M = A \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ m^2 = A \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]
Who is the winner of the tournament?

\[ M = H \begin{bmatrix} 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 \\ J & 0 & 1 & 0 \\ Y & 1 & 0 & 1 \end{bmatrix}, \quad M^2 = A \begin{bmatrix} 0 & 1 & 0 & 0 \\ C & 1 & 0 & 2 \\ J & 1 & 0 & 0 \\ Y & 0 & 1 & 1 \end{bmatrix} \]

\[ M + M^2 = A \begin{bmatrix} A & C & J & Y \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}^2 \]

Cindy won the tournament!

In class:

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**Homework:** Worksheet